4

Organized Comparisons

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4

Organized Comparisons

We have looked at -- and done arithmetic about -- the comparison of values observed separately. Since comparison is so important, many experiments and sets of observations are specifically arranged to be made up of comparisons, rather than separate measurements. We need to learn something about how to look at these sorts of data, how to begin with arithmetic and, where possible, end with display.

4A. Three things compared in pairs

In 1958, Fleckenstein, Freund, and Jackson reported an experiment to compare brands of carbon paper. Their approach was realistic: carbon paper is used in typing, mainly by secretaries and typists. It seemed reasonable to give an individual typist sample material to be typed with each of two brands of carbon paper, asking her to report her preference for one brand over the other on a seven-point scale from -3 to +3.

Giving an individual typist three or more brands seemed likely to yield less precise judgments. If each secretary had been given only one brand, differences from secretary to secretary in the understanding of what specific scores were to mean could not be balanced out. There are many other circumstances where comparison in pairs seems the best that can be done. Comparisons between insect repellants are an obvious example, since nothing is more alike than the two sides of the same cow or the two arms of the same man.

The order in which a secretary typed the test material could influence her report. To balance any such effect, Fleckenstein, Freund and Jackson arranged for the secretaries who compared two specific brands, say C and D, to be divided equally, one subgroup using C first, the other reversing the order -- just as major league baseball teams share home games.

We shall not go into as much detail about the results of this careful experiment as does the published paper, concentrating on the issues that are central for us here. For each pair of brands, 30 secretaries gave preference scores on a numerical scale. Thus there were 30 preference scores that compared this pair of brands directly. Fleckenstein, Freund and Jackson give the details of these preference scores, but it will be enough for us here to use their means. It is of course equivalent and easier to use their sums. (Their midmeans might be at least as interesting.)

Table 1 shows the results for brands C, D, and E. The first panel shows the direct comparisons. We need to remember that +61 for "C minus D" means -(+61) = -61 for "D minus C" and so on.

Suppose that the direct comparison of C with D had been lost. What would we use to replace it? We would still have comparisons of both C and D with E. If C was +13 as compared to E while D was -40, it seems reasonable to form the difference (+13) - (-40) = +53 and say that this tells us something about C as compared to D. The second panel of table 1 gives the three indirect comparisons that this data provides. In forming these, we often need a direct comparison turned around from the direction given in panel (a) since we must follow the pattern

table 1 of chapter 4

Direct Comparisons of Three Brands of Carbon Paper,
And Some Steps Toward Analysis

a) DIRECT COMPARISONS

Brand C minus Brand D: +61

Brand C minus Brand E: +13

Brand D minus Brand E: -40

b) INDIRECT COMPARISONS

C with D: $(+13) - (-40) = +53$

C with E: $(+61) - (-(-40)) = +21$

D with E: $(-(+61)) - (-(+13)) = -48$

c) COMBINED COMPARISONS

C with D: $\frac{(+61) + (+61) + (+53)}{3} = 58.3$

C with E: $\frac{(+13) + (+13) + (+21)}{3} = 15.7$

D with E: $\frac{(-40) + (-40) + (-48)}{3} = -42.7$

Source: Mary Fleckenstein, Richard A. Freund, and J. Edward Jackson 1958. A paired comparison test of typewriter carbon papers. Tappi 41: 128-130.

"U with X" minus "V with X" tells us about "U with V"

In this example, the indirect comparisons are in quite reasonable agreement with the direct comparisons. Clearly we should try to combine them. Equally clearly we trust the direct comparisons somewhat more. Accordingly, we will give each direct comparisons twice the weight of an indirect comparison. The easy way to do this (see Section 1M?) is to add direct comparisons in twice but indirect ones only once. This is done in panel (3).

This particular way of proceeding has a bonus. The direct comparisons were not -- we did not expect them to be -- compatible in detail. We did not expect "C with E" of +13 and "E with D" of +40 to add up to exactly "C with D". Our combined comparisons, however, yield

$$15.7 + 42.7 = 58.4 \quad \text{versus} \quad 58.3$$

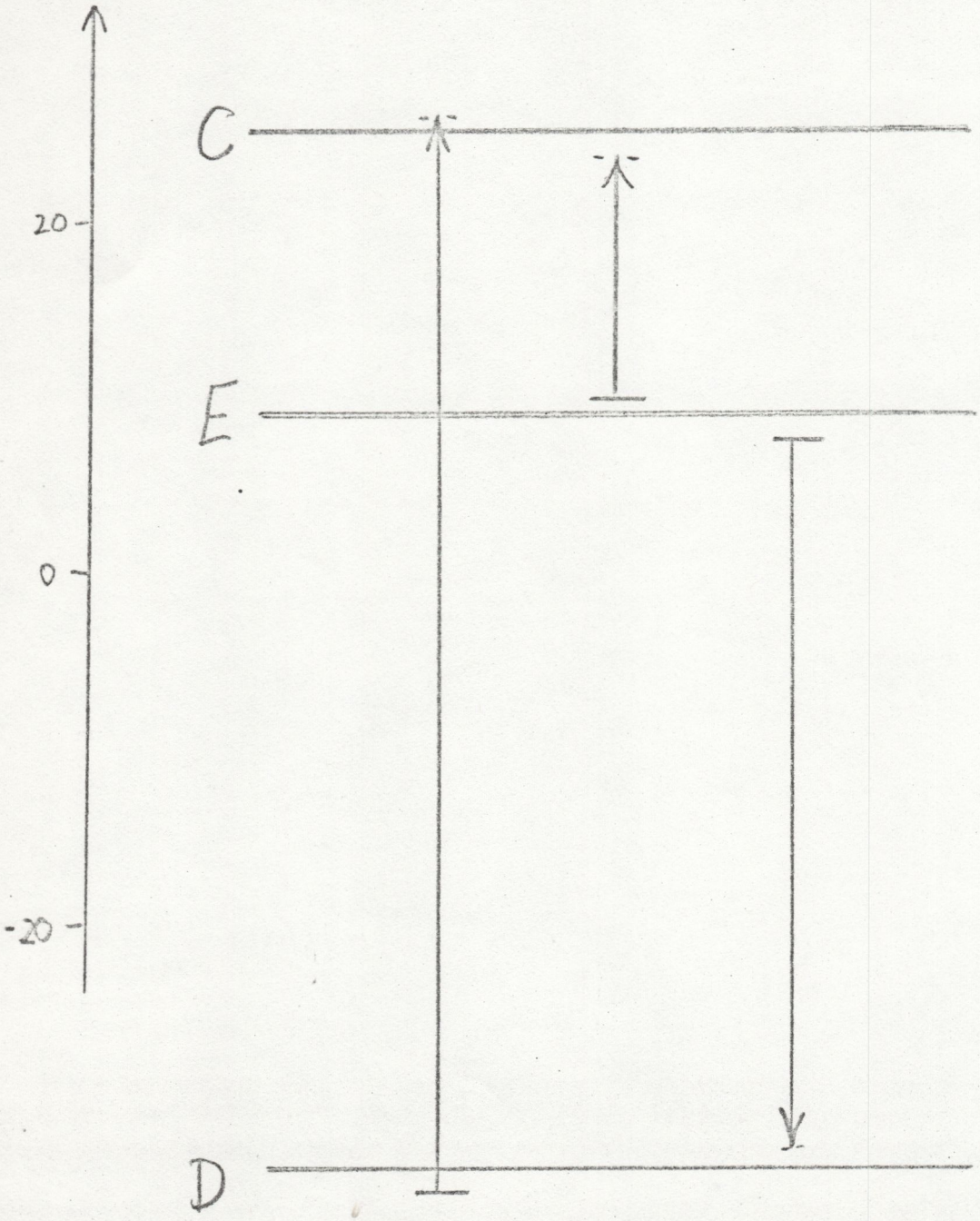
where the discrepancy clearly comes only from forcing thirds into tenths. This is a bonus for this method of analysis.

The value of this bonus, more obvious when we are comparing many more candidates, is that we can represent sets of comparisons when they are compatible with one another, by assigning numbers to the candidates themselves.

Figure 2 shows the fitted positions of the three kinds of carbon paper and the observed differences in relation to the fitted values. We see that the fitted values account quite well for the observed direct comparisons.

plot 2 of chapter 4

Fitted values (horizontal lines) and observed comparisons (arrows) for 3 brands of carbon paper.



In assigning fitted values to the individual brands, we have one free choice. If we were to call C "100" for example, then we would have to use 100 - 58.4 = 41.6 for D and 100 - 15.7 = 84.3 for E. Just how we make this choice does not matter. A convenient choice to make the "scores" for all candidates -- here C, D and E -- add to zero.

It may not be clear that this is a simple choice, but table 3 shows it is. In this table we have laid out all nine comparisons of one of C, D, E with one of C, D, E including "C with C" and both "C with D" and "D with C". Clearly we are to put zero down for "C with C", "D with D" and "E with E". And, as we have already done, we should use minus the result of "C with D" as the result of "D with C".

Summing the columns in table 3 and forming means, we get numbers whose differences are exactly the combined comparisons we saw in table 1. These values were used to make plot 2. Their difference, shown in panel (b) can clearly be used to find residuals. The residuals are shown in panel (c). We specify that they are combining residuals, since they arise from combination and are not usually associated with individual observations. They are relatively small, and have been forced to balance by the structure of the analysis.

The fact that the two residuals having Brand C as "first brand" add to zero (except for rounding) corresponds to the fact that, in plot 2, the observed differences associated with Brand C "balance" around the horizontal line for C. One difference would be better fitted if the line were higher, the other if it were lower. We have done the best we can.

table 3 of chapter 4

The "Square Combining Table" Applied to the Observed Differences Among Carbon Papers

Second brand↓	First brand C	D	E
a) THE DIFFERENCES BETWEEN FIRST BRAND AND SECOND BRAND			
C:	0	-61	-13
D:	+61	0	+40
E:	+13	-40	0
Sums:	+74	-101	+27
Means:	+24.7	-33.7	+9.0
b) THE FITTED DIFFERENCES			
C:	0	-58.4	-15.7
D:	+58.4	0	+42.7
E:	+15.7	-42.7	0
(Sums:)	(+74.1)	(-101.1)	(+27.0)
c) COMBINING RESIDUALS, OBSERVED MINUS FITTED			
C:	0	-2.6	+2.7
D:	+2.6	0	-2.7
E:	-2.7	+2.7	0.0
(Sums)	(-0.1)	(+0.1)	(0.0)

d) EXAMPLE

$$+2.6 = (+61) - (+58.4)$$

4B. Simultaneous comparisons

We do not always have to make comparisons two by two. Often we can compare all candidates together, and then repeat this whole comparison, often several times. How are we to look at such data?

Table 4 sets out some of the results of an experiment on sugar beets reported by Snedecor (1946). The values given are yields in tons per acre for each of three fertilizer treatments applied to a plot in each of six blocks. Land varies. To make precise comparisons we should compare our fertilizer treatments on similar patches -- usually called plots -- of ground. To get similar patches, the best we can do in many situations is get patches that are close together. The simplest way to do this is to divide the field into blocks and then divide each block into a suitable number of plots. Our direct comparisons here are comparisons between plots within a single block.

Panel (a) shows the yields, while panel (b) shows the direct comparisons. Sums and means are given for both. As panel (c) shows, we could find the mean difference between treatments by finding the mean response to each treatment and then taking differences. This works just so long as:

- we want to use means.
- the data is "balanced".

Clearly the next step in this analysis would be to form residuals. We know that we should be wary of residuals from means; we want to use midmeans. The natural thing to do is to take midmeans in either panel (a) or panel (b). Which

table 4 of chapter 4

Yields of Sugar Beets (Tons/Acre) in Each of Six Blocks for Three Amounts of Fertilizer

a) OBSERVED YIELDS

	Fertilizer applied		
	0, None	1, Phosphate only	2, Phosphate and Nitrate
Block 1	2.45	6.71	6.48
Block 2	2.25	5.44	7.11
Block 3	4.38	4.92	5.88
Block 4	4.35	5.23	7.54
Block 5	3.42	6.74	6.61
Block 6	3.27	4.74	8.86
Sum	20.12	33.78	42.48
Mean	3.35	5.63	7.08

b) OBSERVED DIFFERENCES

	"1" minus "0"	"2" minus "1"	"2" minus "0"
Block 1	4.26	-0.23	4.03
Block 2	3.19	1.67	4.86
Block 3	0.54	0.96	1.50
Block 4	0.88	2.31	3.19
Block 5	3.32	-0.13	3.19
Block 6	1.47	4.12	5.59
Sum	13.66	8.70	22.36
Mean	2.28	1.45	3.73

c) CHECKS

$$13.66 = 33.78 - 20.12$$
$$8.70 = 42.48 - 33.78$$
$$22.36 = 42.48 - 20.12$$

also

$$22.36 = 13.66 + 8.70$$

hence

$$2.28 = 5.63 - 3.35$$
$$1.45 = 7.08 - 5.63$$
$$3.73 = 7.08 - 3.35$$

Source: G. W. Snedecor 1946. Statistical Methods, 4th edition. Iowa State College Press, p. 332.

is best, and why? Table 5 shows some of the results, which do not seem to help us to find an answer.

Table 6, involving hypothetical numbers, clears up the main issue. Here the whole comparison has been repeated six times. We will call each set of values to be directly compared with one another a "block" . In doing this, we are using the word "block" in a figurative sense, as any collection of values to be intercompared in the way one compares plots in a block. The left-hand side of this table shows numbers which give differences of 5 in each block. No sensible analysis

table 5 of chapter 4

The Analysis of Table 4 Continued

a) (EXACT) MIDMEANS OF COLUMNS IN (a) OF TABLE 4 AND CORRESPONDING RESIDUALS

	0, None	1, Phosphate only	2, Phosphate and Nitrate
Midmean	3.36	5.50	6.91
Block 1	-.91	1.21	-.43
Block 2	-1.11	-.06	.20
Block 3	1.02	-.52	-1.03
Block 4	.99	-.27	.63
Block 5	.06	1.24	-.30
Block 6	-.09	-.76	1.95

b) (EXACT) MIDMEANS OF COLUMNS IN (b) OF TABLE 4 AND CORRESPONDING RESIDUALS

	"1" minus "0"	"2" minus "1"	"2" minus "0"
Midmean	2.29	1.24	3.75
Block 1	1.95	-1.47	0.28
Block 2	.90	.43	1.11
Block 3	-1.75	-.28	-2.25
Block 4	-1.41	1.07	-.56
Block 5	1.03	-1.37	-.56
Block 6	-.82	2.88	1.84

table 6 of chapter 4

Hypothetical Tables Illustrating Why We Want Midmean Differences

	Original		Modified	
a) "OBSERVED" VALUES				
	A	B	A	B
Block 1	10	5	10	5
Block 2	9	+4	9	-4
Block 3	5	0	5	0
Block 4	0	-5	0	-5
Midmean	7	2	7	-2
b) "OBSERVED" DIFFERENCES "A minus B"				
Block 1	5		5	
Block 2	5		13	
Block 3	5		5	
Block 4	5		5	
Midmean	5		5	
c) COMPARISON OF VALUES OF "A MINUS B" FOUND FROM MIDMEANS				
From (a)	5 = 7-2		9 = 7 - (-2)	
From (b)	5		5	

can give any overall comparison except 5. On the right-hand side, one "observed" number has been changed from +4 to -4. The result is a change in the midmean for that treatment but not in the midmean of the differences. If we want our analysis to be resistant to such changes, we need to work with differences.

If we are going to use midmeans, then, panel (b) of table 5 is the relevant panel. The indicated differences are

"1" minus "0": 2.29

"2" minus "1": 1.24

whose sum is 3.53, but

"2" minus "0": 3.75

Even though all comparisons were made in each block, the raw results of the midmean analysis do not give us the bonus of perfect compatibility. We know how to get this bonus, however. We need only use the square combining table. Table 7 shows the computations and the result. Clearly the fit obtained by the square combining table is again a good one.

If we want to represent the midmean comparisons of the various fertilizers as differences among values for treatments, it seems that the best we can do is

0, No fertilizer added: down 2.01

1, Phosphate only: up 0.35

2, Phosphate and nitrate: up 1.66

table 7 of chapter 4

The Square Combining Table Applied to the Midmean Differences

(See table 5, panel (b) for input)

Second treat.↓	0, None	1, Phosphate only	2, Phosphate and nitrate
a) FIRST TREATMENT MINUS SECOND TREATMENT			
0	0	2.29	3.75
1	-2.29	0	1.24
2	-3.75	-1.24	0
Sum	-6.04	1.05	4.99
Mean	-2.01	.35	1.66
b) FITTED DIFFERENCES			
0	0	2.36	3.67
1	-2.36	0	1.31
2	-3.67	-1.31	0
c) COMBINING RESIDUALS			
0	0	-.07	+.08
1	+.07	0	-.07
2	-.08	+.07	0

where these ups and downs are from a rather arbitrary zero. This arbitrariness is of little concern to us, so long as our concern is only with actual comparisons.

It is easy to be naive enough to feel that enough direct comparisons might eliminate any need for midmeans. That this is not so, even in the very simplest case where there is no need for the square combining table, can be seen from an experiment of Mitchell, Burroughs and Beadles on the biological value of proteins for peanuts, as shown by rat growth. Table 8 sets out the data, with means and midmeans. Here, as a look at the wings, hinges, and body layout for the differences will convince almost anyone, the midmean difference of 3.6 is a more useful summary of the results than any other combination of means or midmeans.

4C. <u>The full carbon-paper experiment -- fuller treatment</u>

As we might suspect from the notation C, D, E above, Fleckenstein, Freund, and Jackson actually ran their experiment on five brands of carbon paper (or more). Table 9 summarizes, in square combining table form, the results for all $\frac{1}{2}(5)(4) = 10$ comparisons, and proceeds to find fitted values and residuals.

When we look at the residuals -- every column of which must sum to zero because of the way we have done the arithmetic -- we have an impression of unsymmetrical straggling. In order to take this further, the last line of table 9 contains the midmeans of the columns of residuals. There continues to be a suggestion, not too strong, that we might be wise to consider taking account of this appearance.

table 8 of chapter 4

Biological Value of Proteins in Raw and Roasted Peanuts as Calculated From Rat Growth

Pair of rats	Biological values		
	Raw	Roasted	Difference
1	61	55	6
2	60	54	6
3	56	47	9
4	63	59	4
5	56	51	5
6	63	61	2
7	59	57	2
8	56	54	2
9	44	62	-18
10	61	58	3
Mean	57.9	55.8	2.1
Midmean	58.9	56.1	3.6

Source: H. H. Mitchell, Wise Burroughs, Jesse R. Beadles 1936. ---------- Journal of Nutrition, 11, 257.

table 9 of chapter 4

Analysis of the Full Carbon-Paper Experiment

Second brand	First brand A	B	C	D	E
a) SUMS OF DIFFERENCES IN SCORES					
A	0	-13	+36	-41	-2
B	+13	0	+21	-20	+13
C	-36	-21	0	-61	-13
D	+41	+20	+61	0	+40
E	+2	-13	+13	-40	0
Sum	+20	-27	+131	-162	+38
Mean	+4.0	-5.4	+26.2	-32.4	+7.6
b) FITTED DIFFERENCES					
A	0.0	-9.4	+22.2	-36.4	3.6
B	9.4	0.0	31.6	-27.0	13.0
C	-22.2	-31.6	0.0	-58.6	-18.6
D	36.4	27.0	58.6	0.0	40.0
E	-3.6	-13.0	18.6	-40.0	0.0
c) COMBINING RESIDUALS					
A	0	-3.6	13.8	-4.6	-5.6
B	3.6	0	-10.6	7.0	0.0
C	-13.8	10.6	0	-2.4	5.6
D	4.6	-7.0	2.4	0	0.0
E	5.6	0.0	-5.6	0.0	0
Midmean	2.8	-1.1	-1.0	-0.7	0.0

These midmean residuals offer a convenient way to do just that. The original fit to brand A was +4.0. The midmean combining residual was +2.8. Accordingly it would be natural to try a fit of

$$+6.8 = +4.0 + 2.8$$

for brand A, and corresponding values for the other brands. Table **10** carries this out, producing new fits and new residuals. This time the midmean residuals are substantially smaller, and we are inclined to trust the fitted values further, especially for the definition of residuals.

Clearly, we could repeat this process as many times as we wish. It is probably best, as a routine procedure, to do it only once. Exceptional situations may require more, but this should not be a routine procedure -- every such instance should be examined from a number of points of view, and thought through carefully. (Experience may lead to relaxing this position of caution.)

We believe the residuals of 10(c) are better than those of 9(c). Let us look at them. Plot **11** displays their values, omitting all those that must be 0 because they are on the diagonal. As is so likely to happen, this plot tends to focus our attention on the outer parts of the batches, mainly reminding us that the largest residuals are larger for the fuller treatment.

So that we may be clearer about what is going on, plot **12** shows the schematic summaries for the same two sets of

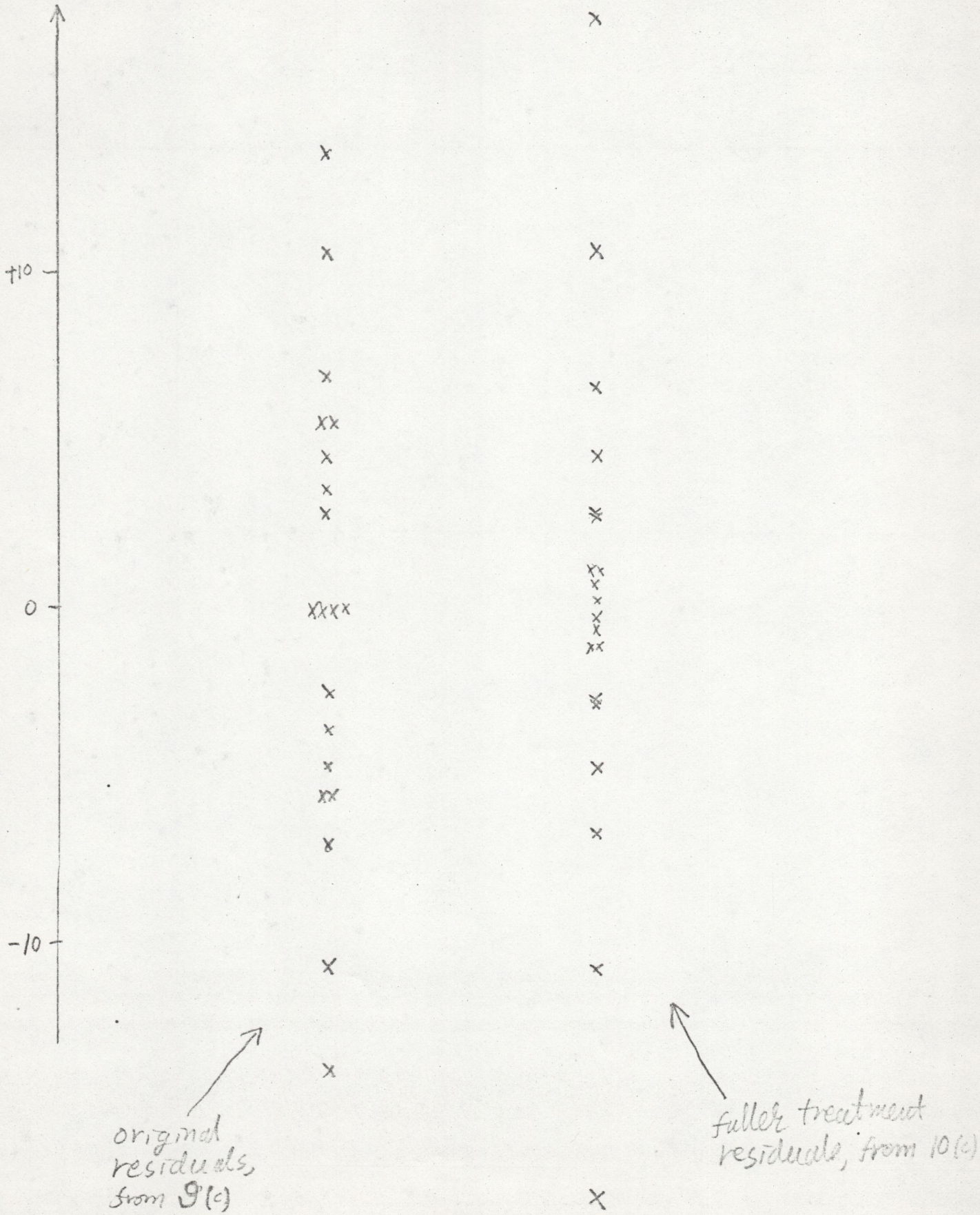
table 10 of chapter 4

Fuller Treatment of the Data of Table 9

	First brand				
	A	**B**	**C**	**D**	**E**
a) CALCULATION OF NEW FIT					
Means from 9(a)	+4.0	-5.4	+26.2	-32.4	+7.6
Midmeans from 9(c)	+2.8	-1.1	-1.0	-0.7	0.0
Sums	+6.8	-6.5	+25.2	-33.1	+7.6
b) FITTED DIFFERENCES (BASED ON NEW FIT)					
A	0	-13.3	18.4	-39.9	+0.8
B	13.3	0	31.7	-26.6	+14.1
C	-18.4	-31.7	0	-58.3	-17.6
D	39.9	26.6	58.3	0	40.7
E	-0.8	-14.1	17.6	-40.7	0
c) COMBINING RESIDUALS					
A	0	+0.3	17.6	-1.1	-2.8
B	-0.3	0	-10.7	6.6	-1.1
C	-17.6	10.7	0	-2.7	4.6
D	1.1	-6.6	2.7	0	-0.7
E	2.8	1.1	-4.6	0.7	0
Midmean	0.2	0.4	-0.6	-0.1	-0.6

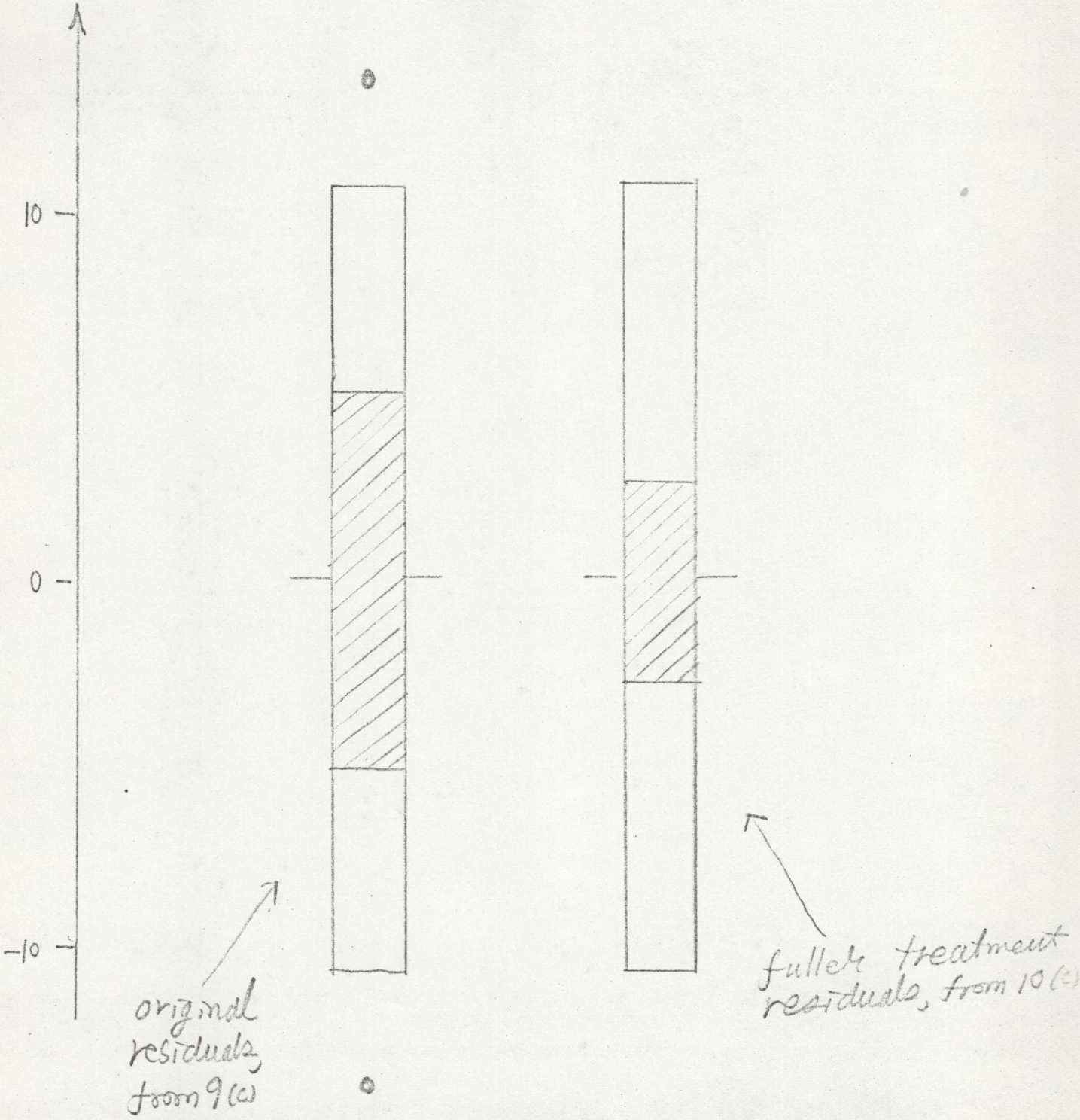
plot 11 of chapter 4

The two sets of residuals



plot 1L of chapter 4

Schematic summaries of the two sets of residuals



residuals. The increase in size of the largest residuals is still clear, but even clearer is the large shrinkage of the (easy) midspread, actually from 10.2 to 5.5. If midspread of residuals is a useful criterion of closeness of partial description, as we tend to believe, the second set of residuals is very much better than the firs

4D. The intermediate case

We have looked at two extremes:

- only comparisons in pairs.
- all candidates compared simultaneously.

Obviously there can be an intermediate situation where each planned comparison involves more than two but less than all of the candidates. Clearly we can apply to that situation the process we have been evolving step by step, namely:

1) Look at all occurrences of any one comparison, note the corresponding differences, form the midmeans.

2) Enter these midmeans -- and their negatives -- in a square combining table, find fitted differences, combining residuals, and midmean combining residuals.

3) If the midmean residuals seem large enough to concern us, add them to the original fits to form a new set of fits, and complete the process of fuller treatment by finding new fitted differences and new combining residuals.

Table 13 shows a part of data reported by Eden and Fisher on the yield of straw and grain under each of eight manurial treatments. In this illustration, the incompleteness of comparison has been enforced by our selection of data. The

table 13 of chapter 4

Yields of Straw Under Eight Different Manurial Treatments

a) DATA

	Treatment							
Block	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
1	242	-	-	-	322	200	260	203
2	321	382	-	-	-	261	318	275
3	261	201	298	-	-	-	266	207
4	317	316	381	255	-	-	-	331
5	255	280	300	238	232	-	-	-
6	-	285	294	309	393	258	-	-
7	-	-	256	283	351	306	276	-
8	-	-	-	324	363	376	385	328

b) DIFFERENCES, AND THEIR (EXACT) MIDMEANS

A-B	A-C	A-D	A-E	A-F	A-G	A-H
-61	-37	62	-80	42	-18	39
60	-64	17	23	60	+3	46
1	-45				-5	54
-25						-14
-12	-45	40	-28	51	-6	42

B-C	B-D	B-E	B-F	B-G	B-H	C-D
-97	61	48	121	64	107	126
-65	42	-108	27	-65	-6	62
-20	-24				-15	-15
-9						-27
-42	34*	-30	74	0	11	24

C-E	C-F	C-G	C-H	D-E	D-F	D-G
68	36	32	91	6	51	7
-99	-50	-20	50	-84	-23	-61
-95				-68	-52	
				-39		
-70	-7	6	70	-54	-16	-27

table 13 (Cont'd.) - 2

D-H	E-F	E-G	E-H	F-G	F-H	G-H
-76	122	62	119	-60	-3	57
-4	135	75	35	-57	-14	43
	45	-22		30	48	59
	-13			-9		57
-40	84	50	77	-33	4	57

Minus treatment	Plus treatment A	B	C	D	E	F	G	H
c) SQUARE COMBINING TABLE								
A	0	+12	45	-40	28	-51	6	-42
B	-12	0	42	-34	30	-74	0	-11
C	-45	-42	0	-24	70	7	-6	-70
D	40	34	24	0	54	16	27	40
E	-28	-30	-70	-54	0	-84	-50	-77
F	51	74	-7	-16	84	0	33	-4
G	-6	0	6	-27	50	-33	0	-57
H	42	11	70	-40	77	4	57	0
Sum	42	59	110	-235	393	-215	67	-221
Mean*	5	7	14	-30	49	-27	8	-28
d) FITTED DIFFERENCES								
A	0	2	9	-35	44	-32	3	-33
B	-2	0	7	-37	42	-34	1	-35
C	-9	-7	0	-44	35	-41	-6	-42
D	35	37	44	0	79	+3	38	2
E	-44	-42	-35	-79	0	-76	-41	-77
F	32	34	41	-3	76	0	35	-1
G	-3	-1	6	-38	41	-35	0	-36
H	33	35	42	-2	77	1	36	0
e) FIRST COMBINING RESIDUALS (WEIGHTS IN PARENTHESES)								
A	0(3)	10(4)	36(3)	-5(2)	-16(2)	-19(2)	3(3)	-9(4)
B	-10(4)	0(3)	35(4)	3(3)	-12(2)	-40(2)	-1(2)	24(3)
C	-36(3)	-35(4)	0(3)	20(4)	35(3)	48(2)	0(2)	-28(2)
D	5(2)	-3(3)	-20(4)	0(3)	-25(4)	13(3)	-11(2)	38(2)
E	16(2)	12(2)	-35(3)	25(4)	0(3)	-8(4)	-9(3)	0(2)
F	19(2)	40(2)	-48(2)	-13(3)	8(4)	0(3)	-2(4)	-3(3)
G	-3(3)	1(2)	0(2)	11(2)	9(3)	2(4)	0(3)	-20(4)
H	9(4)	-24(3)	28(2)	-38(2)	0(2)	3(3)	20(4)	0(3)
Weighted midmeans*	+1	-1	+3	+6	+1	0	0	-4

table 13 (Cont'd.) - 3

	A	B	C	D	E	F	G	H
f) WEIGHTED FULLER-TREATMENT FITS								
means(c)	5	7	14	-30	45	-22	8	-28
midmeans(e)	1	-1	3	6	1	0	0	-4
sums	6	6	17	-24	46	-22	8	-32
g) NOTES AND EXAMPLES FOR (e)								
Unweighted midmeans	+3	+2	+2	+2	-1	-1	-1	-3

Weighted midmean for B:

Sum of weights 4+3+4+3+2+2+2+3 = 23 = 5 + 1 + 11 + 1 + 5

wing	40, 40, 12, 12, 10
hinge	10
body	10, 10, 1, 1, 0, 0, 0, -3, -3, -3, -24
hinge	-24
wing	-35, -35, -35, -35, -24

Sum of body = -11, mean of body = -1.0, rounded = -1

Source: T. Eden and R. A. Fisher 1927. Studies in Crop Variation. IV. The experimental determination of the value of top dressings with cereals. Journal of Agricultural Science 17: 548-562.

problem of analysis, however, is not altered by the way in which the incompleteness comes about. Panels (b), (c) and (d) of table 13 carry the analysis through the collection of differences, the formation of the square combining table and the calculation of fitted differences. As we collect differences, it is clear that some comparisons appear four times, some three, and some only twice. While we are likely to want to make some allowance for this in due course, the essential feature of the square combining table -- differences of means used to get means of differences -- would make it hard for us to take account of these differences in weight at that point. Accordingly, we delay any account of weights.

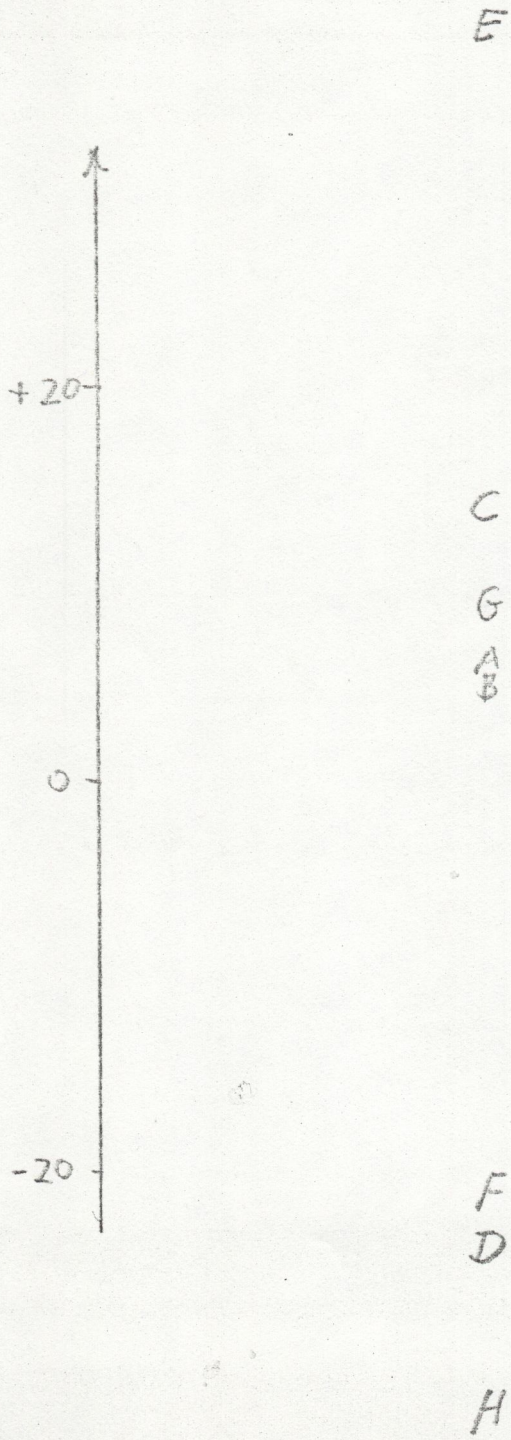
Panel (e) presents the (first) combining residuals, and at this point we note (in parenthesis) with each residual the number of comparisons from which it comes. Then, when it is time to take midmeans of columns of residuals for fuller treatment, we can form weighted midmeans. This is done at the close of (e), an example being given in (g).

Panel (f) combines the weighted midmeans from (e) with the original means from (c). The resulting fuller-treatment values are not enough different from the first stage values to make us insist on a second set of combining residuals at this point. (We shall find it worthwhile to do this in a moment.)

Plot 14 shows the fitted values visually. A reasonable verbal summary of the appearances would be:

plot 14 of chapter 4

Relative amounts of straw by treatment
(as filled in table 13(G))



- high yield of straw: E
- intermediate yield of straw: C,G,A,B
- low yield of straw: F,D,H

Bear in mind that this is just a summary of appearances, and has not been subjected to confirmatory analysis.

Before passing on to the next topic, we should emphasize that the symmetry in pattern of the straw yield data made no noticeable difference in the way it was analyzed. If, for example, we had lost -- or chosen to leave out -- almost any five values among the original data the whole process would have gone through according to the same pattern. We would have had fewer comparisons, and, when we came to weighting, our weights would not have been neatly constant along diagonals -- of course our results would not be quite the same -- but both method and its reasonableness would have been unaffected.

4E. Individual residuals

So far in this chapter, the only residuals we have formed and looked at have been residuals that have to come about in the process of combination. So long as we cannot look behind the entries in the square combining table -- either because only one comparison contributes to one entry or because we do not have access to the individual comparisons -- this is all that we can do. Otherwise, we can -- and usually should -- go back to the original comparisons.

Let us go back to the yields of sugar beets, as given in table 4, and apply the final fitted values obtained in table 7, namely -2.01, 0.35, and 1.66, to the observed values. Table 15 shows the data and results. The first step is to try

table 15 of chapter 4

The Sugar Beet Data Again: Calculation of Individual Residuals

Block	0, none	1, phosphate	2, phosphate and nitrate	
a) OBSERVED VALUES (table 4)				
1	2.45	6.71	6.48	
2	2.25	5.44	7.11	
3	4.38	4.92	5.88	
4	4.35	5.23	7.54	
5	3.42	6.74	6.61	
6	3.27	4.74	8.86	
b) FITTED ADJUSTMENTS (table 7, mean forced to 0)				
	-2.01	0.35	1.66	
c) ADJUSTED VALUES (observed minus adjustments)				
				(*)
1	4.46	6.36	4.82	(5.01)
2	4.26	5.09	5.45	(5.01)
3	6.39	4.57	4.22	(4.82)
4	6.36	4.88	5.88	(5.79)
5	5.43	6.39	4.95	(5.51)
6	5.28	4.39	7.20	(5.45)

(*) Midmeans by block.

d) INDIVIDUAL RESIDUALS (adjusted values minus midmeans)			
1	-.51	1.35	-.19
2	-.75	.08	.44
3	1.57	-.25	-.60
4	.57	-.91	.09
5	-.08	.88	-.56
6	-.17	-1.06	1.75

Note: Only +1.75 is "far out".

to make the values for the different treatments in a block reasonably comparable by subtracting the corresponding fitted values. Once this far, it is easy and natural to find midmeans of these adjusted values, one for each block, and then subtract these midmeans to form individual residuals.

Exactly the same process can be followed when the original data is incomplete. Table 16 illustrates this on the 5/8 of the Eden and Fisher data on straw yields discussed in the last section. This time the individual residuals show more character, 5 out of 40 being "far out" according to the simple procedure. Just how this fact is to be interpreted here would take more knowledge than we have of the original data, its source, and -- probably -- the agriculture of straw yields in British grain farming. The standard point is clearly made, however;

- if you do not calculate residuals, you will not learn what they can tell you.

4F. <u>How far have we come</u>?

The central ideas of this chapter are four:

- comparisons of one with another, and their combination.
- blocks, as a natural element of data taking.
- the need, and schemes, for making comparisons compatible.
- even with incomplete comparisons in blocks, we can find a residual for each individual observation.

table 16 of chapter 4

The Straw Yield Data Again: Calculation of Individual Residuals

Block	A	B	C	D	E	F	G	H	
	Manurial treatment								
a) OBSERVED VALUES (table 13(a))									
1	242	-	-	-	322	200	260	203	
2	321	382	-	-	-	261	318	275	
3	261	201	298	-	-	-	266	207	
4	317	316	381	255	-	-	-	331	
5	255	280	300	238	232	-	-	-	
6	-	285	294	309	393	258	-	-	
7	-	-	256	283	351	306	276	-	
8	-	-	-	324	363	376	385	328	
b) FITTED ADJUSTMENTS (table 13(f))									
	6	5	17	-24	39	-21	9	-32	
c) ADJUSTED VALUES (observed minus adjustment)									(*)
1	236	-	-	-	283	221	251	235	(240)
2	315	377	-	-	-	282	309	307	(310)
3	255	196	281	-	-	-	257	239	(251)
4	311	311	364	279	-	-	-	363	(312)
5	249	275	283	262	193	-	-	-	(262)
6	-	280	277	333	354	279	-	-	(296)
7	-	-	239	307	312	327	267	-	(296)
8	-	-	-	348	324	397	376	360	(361)

(*) = midmeans by block.

d) INDIVIDUAL RESIDUALS (adjusted minus midmeans)

1	-4	-	-	-	43	-19	11	-5
2	5	67	-	-	-	-28	-1	-3
3	4	-55	30	-	-	-	6	-12
4	-1	-1	52	-33	-	-	-	52
5	-13	13	21	0	-69	-	-	-
6	-	-16	-19	37	58	-17	-	-
7	-	-	-57	11	16	33	-29	-
8	-	-	-	-13	-37	36	15	-1

Note: The following are "far out": 67,-55,-57,-69,58, while 52 and 52 are nearly so.

Most data gathering is for comparison -- either there-and-then or with more distant results. Comparison within the given body of data is very common and very important. Good ways to do it are needed. Comparisons, in the first instance, are of one with another, yet comparison among several is something we do frequently. Convenience and compactness are served when we can assign a single number to each candidate or treatment such that the difference of two such numbers is a good comparison of the two corresponding candidates or treatments. Such a set of numbers is the natural and desirable partial description of these candidates or treatments for the comparison of each with all others.

Comparisons are always under circumstances. These circumstances often change recognizably from one part of our body of data to another. Precision of comparison is higher if the data yielding numbers to be differenced came from one set of circumstances. Comparisons are thus likely to be made within blocks . Blocks can offer us both increased precision of comparison and increased diversity of circumstance.

While it is good to have a single number for each candidate or treatment -- numbers whose differences tell us the comparisons -- we do not get such a set of numbers without effort. Even direct comparisons of all pairs among as few as three candidates or treatments are not going to be exactly compatible when separately measured.

A convenient and effective way to enforce compatibility is the square combining table. In certain instances,

not too frequent, it is quite useful to polish up the results of using the square combining table. The process called fuller treatment can add a useful polish in many of these circumstances.

While the square combining table generates combining residuals (a necessary step on the road to fuller treatment), it does not generate individual residuals. Individual residuals can, however, be found with not too much extra effort.

*****

4K. How to subtract

All of us know how to subtract. There is nothing essentially new that I can tell you about subtraction. Yet matters of minor technique seem to provide gains in both speed and ease. This is especially true when we are subtracting a single mean or midmean from all the entries in a column or row as we so frequently have to in forming first or second residuals.

Back in Chapter 1, the mean of the 10 bowling scores was, rounded off, 198 so that we needed to subtract this, in turn, from 193, 214, 185, 198, 203, 207, 191, 184, 210, 192. How, in detail, should we do this? The differences are all small, surely our mental arithmetic should be equally simple.

We start with 193 - 198, for which the answer is obviously negative. Subtracting 193 from 198 obviously gives 5. So the first residual is -5.

Next 214. The residual 214 - 198 is obviously positive. How are we to find its size easily? The difference between subtracting 98 and adding 2 is 100, one way or the other. Thus 14 + 2 = 16 gives the last two figures of our residual. It is clear that this residual is not 116 (and not -84) so +16 must be the answer.

Steaming ahead, 185 is easy; since 98 - 85 = 13, it gives -13. 198 is easier still; its residual is 0. 203 is easy; since 3 + 2 = 5 it gives +5. The remaining residuals

are easy, coming from 7 + 2 = 9, 98 - 91 = 7, 98 - 84 = 14, 10 + 2 = 12, 98 - 92 = 6.

You may justly say that this example was extreme in two ways: (i) 98 led to 2, which is an unusually small number, (ii) we find small residuals of big numbers. But it would be wrong to say or feel that the same device will not work and work effectively in very different instances. Let us look at some.

```
        12.74
less     9.27
        ------
gives   -.--
```

We clearly can subtract 27 from 74 without borrowing, so the first figure in the answer is 3, to be followed by 74 - 27 = 47. Answer: 3.47.

```
        12.27
less     9.74
        ------
gives   -.--
```

This time we must borrow one, making the left-hand figure 3 - 1 = 2. Subtracting 74 differs from adding 26 by a multiple of 100. We have already taken care of this, so we need only form 27 + 26 = 53. Answer: 2.53.

In hand data analysis, we work most frequently with 2- or 3-figure numbers and sometimes with 4-figure numbers. (1-figure numbers, if appropriate, make arithmetic quite easy. 5- or more figure numbers are not often needed, particularly after the formation of first residuals). In this range the

device of replacing subtractions by addition, throwing in or out a count of 100 in the least significant figure is routinely and regularly helpful.

This can even be true when the two sets of last two figures are in easy order. Thus

$$\begin{array}{rr} & 12.71 \\ \text{less} & \underline{9.48} \\ \text{gives} & -.-- \end{array}$$

To some, 71 - 48 = 23 is quite clear. To others 71 + 52 = 123 is an easier way to reach 23.

If we want to take a "refined" attitude in saying what we have been doing, it is easy to do so. We have only to say that we have been doing arithmetic to the base 100 rather than to the base 10. This remark carries much more truth than help.

The important fact is that we are trying to make things simple for an individual. Some of us will find this method of operating with 2-figure blocks helpful. Those who do should use this device. Others will find 1-figure-at-a-time easier. They should use it. (There may be some, though I cannot recall meeting them, who work best three figures at a time.) What is important, however, is trying 2-figures-at-a-time enough to find out whether it will be easier for you.

*****

4P. <u>IEDA processor</u>

*****

4R. Where the square combining table has to wait

If the available comparisons are too few in kind, we cannot apply the square combining table to our direct comparisons. Instead we have to combine direct and indirect comparisons more tediously. And then, since the results of these combinations are NOT automatically compatible, we need to use the square combining table to enforce compatibility.

Table 17 takes the remainder of the Eden and Fisher data and carries out the process.

Only a few points need attention. We have -- arbitrarily so far as this account is concerned -- decided to weight indirect comparisons through direct comparisons of weights w_1 and w_2 with weight

$$\frac{2w_1w_2}{w_1+w_2}$$

Thus an indirect comparison through weights 1 and 2 gets weight 4/3, intermediate between weight 1, for two links of weight 1, and weight 2, for two links of weight 2, as it should be. The weights for the comparison of a treatment with itself is again set at the mean of the other weights in the row.

Clearly there is some interest in comparing the results found here for 3/8 of the original data with those found in section 4D for the other 5/8. Table 18 makes the comparison. The agreement, as we might expect, is far from

table 17 of chapter 4

The Other Part of the Eden and Fisher Data and Its Analysis

a) THE DATA

	Manurial treatment							
Block	A	B	C	D	E	F	G	H
1	-	267	215	212	-	-	-	-
2	-	-	330	292	370	-	-	-
3	-	-	-	265	284	259	-	-
4	-	-	-	-	323	361	340	-
5	-	-	-	-	-	234	362	229
6	331	-	-	-	-	-	400	266
7	216	200	-	-	-	-	-	276
8	295	295	284	-	-	-	-	-

b) THE DIRECT COMPARISONS

A-B	A-C	A-G	A-H	B-C	B-D	B-H	C-D
16	11	-69	65	52	55	-76	3
0			-60	11			38
8	11	-69	2	32	55	-76	20

C-E	D-E	D-F	E-F	E-G	F-G	F-H	G-H
-40	-78	6	25	-17	21	5	133
	-19		-38		-128		134
-40	-98	6	-6	-17	-54	5	134

c) THE DIRECT AND INDIRECT COMPARISONS (WITH WEIGHTS)

A with B

direct	8	(2·2 = 4)
via C	-21 = 11 - 32	(4/3)
via H	84 = 8 - (-76)	(4/3)
wt'd sum*	116	
wt'd mean*	17	

A with C

direct	11	(2·1 = 2)
via B	40 = 8 - (-32)	(2)
	102	
	+26	

A with D

via B	63 = 8 - (-55)	(4/3)
via C	31 = 11 - (-20)	(4/3)
wt'd sum	125+	
wt'd mean	47	

A with E

via C	-29 = 11 - (40)	(1)
via G	-42 = - 69 - (-17)	(1)
	-71	
	-36	

(The other 24 comparisons go in the same ways, the results are summarized in (d), following.)

table 17 (Cont'd.) - 2

d) SQUARE COMBINING TABLE FOR COMBINED COMPARISONS

	Plus treatment							
Minus	A	B	C	D	E	F	G	H
A	0	-17	-26	-47	36	6	98	0
B	17	0	-27	-54	0	5	144	38
C	26	27	0	-18	34	10	52	56
D	47	54	18	0	38	24	56	60
E	-36	0	-34	-38	0	-15	38	76
F	-6	-5	-10	-24	15	0	62	-38
G	-98	-144	-52	-56	-38	-62	0	-108
H	0	-38	-56	-60	-76	38	108	0
Sums	-42	-123	-187	-297	7	6	458	74
Means	-6	-15	-23	-37	1	1	57	9

e) FIRST FITTED DIFFERENCES

A	0	-9	-17	-31	7	7	63	15
B	9	0	-8	-22	16	16	72	24
C	17	8	0	-14	24	24	80	32
D	31	22	14	0	38	38	94	46
E	-7	-16	-24	-38	0	0	56	8
F	-7	-16	-24	-38	0	0	56	8
G	-63	-72	-80	-94	-56	-56	0	-48
H	-15	-24	-32	-46	-8	-8	48	0

f) FIRST SQUARE-TABLE RESIDUALS (3 TIMES WEIGHTS IN PARENTHESES)

A	0(11)	-8(20)	-9(12)	-16(8)	29(6)	-1(8)	25(12)	-15(20)
B	8(2)	0(11)	-19(20)	-32(12)	-16(8)	-11(6)	72(8)	14(12)
C	9(12)	19(20)	0(11)	-4(20)	10(12)	-14(8)	-28(6)	24(8)
D	16(8)	32(12)	4(20)	0(11)	0(20)	-14(12)	-38(8)	14(6)
E	-29(6)	16(8)	-10(12)	0(20)	0(11)	-15(20)	-18(12)	68(8)
F	1(8)	11(6)	14(8)	14(12)	15(20)	0(11)	+6(20)	-46(12)
G	-35(12)	-72(8)	28(6)	38(8)	18(12)	-6(20)	0(11)	-60(20)
H	15(20)	-14(12)	-24(8)	-14(6)	-68(8)	+46(12)	60(20)	0(11)
wt'd midmeans	+7	+5	-5	-2	+7	-9	+12	-11

g) FULLER TREATMENT FITTED VALUES

means (d)	-6	-15	-23	-37	1	1	57	9
midmeans (f)	7	5	-5	-2	7	-9	12	-11
sums	1	-10	-28	-39	8	-8	69	-2

table 18 of chapter 4

Comparison of Results for Different Parts
of the Eden and Fisher Data

	Manurial treatment							
	A	B	C	D	E	F	G	H
5/8 (in 4D)	6	5	17	-24	39	-21	9	-32
3/8 (here)	1	-10	-28	-39	8	-8	69	-2
wt'd mean	4	-1	0	-30	27	-16	32	-21

perfect. It is good enough, however, to suggest that confirmatory data analysis would cause us to give a small but definite amount of credence to the results. In particular, treatment D seems to produce less straw than the average of the others.

We need to say again that we have not claimed -- and would not claim -- that the analysis set out here is "the best" or "the most precise".

At our stage of experience, we should be happy to have one way of analyzing such data that is moderately reasonable.

*****

4Y. Problems

1. Tucker, Wakeley and Cochran (1951) determined the effect of washing and removing excess moisture by wiping or by air current on the ascorbic acid content of turnip greens. The data are given in table 19. Analyze and comment.

2. Bing (1953) compared the effect of several herbicides on the weights of the spikes produced by flowering gladiolus plants. Table 20 gives the data. Analyze and comment.

3. Sherwood (1962) studied the influence of type of soil on the development of strength in mixtures of soil and cement. Table 21 gives some of his data. Analyze the seven-day and 28-day data separately. Discuss the comparison.

4. Youden (1951) reports on the comparison of seven thermometers in sets of three. His description deserves attention as an example of intelligent care. Table 22 gives the results after correction for time between readings. Analyze and comment.

5. Seliger (1950) reported on a more effective way of comparing radioactive standards adopted by the National Bureau of Standards (namely using blocks). Table 23 gives some of his data. Analyze and comment.

6. Gulliksen and Tukey (1958), in discussing the confirmatory analysis of data consisting of paired comparisons among seven specimens of handwriting (taken from the Ayres scale). Table 24 summarizes the judgments made by two groups,

table 19 of chapter 4

Milligrams of Ascorbic Acid Per 100 Grams
Dry Weight of Turnip Greens

Block	Control	Washed and blotted dry	Washed and dried in air current
1	950	857	917
2	887	1189	1072
3	897	918	975
4	850	968	930
5	975	909	954

Source: H. P. Tucker, J. J. Wakeley, and F. D. Cochran 1951. Effect of washing and removing excess moisture by wiping or air current on the ascorbic acid content of turnip greens. Southern Cooperative Service Bulletin 10: 54-56.

See also: R. G. D. Steel and J. H. Torrie 1960. Principles and Procedures of Statistics, page 137.

table 20 of chapter 4

Average Weight of Gladiolus Spikes in Ounces

Block	2.4-D TCA	Check	DN/CR	Sesin
1	2.05	1.25	1.95	1.75
2	1.56	1.73	1.00	1.93
3	1.68	1.82	1.83	1.70
4	1.69	1.31	1.81	1.59

Source: A. Bing 1954. Gladiolus control experiments, 1953. The Gladiolus 1954 (New England Gladiolus Society, Inc.)

See also: R. G. D. Steel and J. H. Torrie 1960. Principles and Procedures of Statistics, page 137.

table 21 of chapter 4

Unconfined Compressive Strength (in Pounds Per Square Inch) of 10% Cement - 90% Soil Mixtures After 7 and 28 Days

	After 7 days				After 28 days			
Batch	I	II	III	IV	I	II	III	IV
A	490	520	590	810	830	1020	970	1500
B	380	460	400	620	760	870	680	1210
C	345	45	20	395	720	750	860	1330
D	295	75	250	550	660	770	670	1040
E	380	60	20	90	670	940	620	1000
F	350	220	430	540	640	800	845	930
G	410	80	175	70	860	870	770	1130
H	335	265	500	905	750	980	830	1270

I = Leighton Buzzard sand

II = Tunstall Common A

III = Tunstall Common B

IV = Whitchurch

Source: P. T. Sherwood 1962. Rapid method for detecting the presence of deleterious organic matter in soil-cement. Journal of Applied Chemistry 12: 279-288.

See also: R. L. Wine 1964. Statistics for Scientists and Engineers.

table 22 of chapter 4

Adjusted (For Time of Reading) Temperatures
Read on Seven Thermometers

(Entries Are Thousandths of a Degree Above 30.000°C)

Set	Tags and readings		
1	A 56	B 31	D 35
2	A 58	E 16	F 41
3	B 41	C 53	E 24
4	B 46	F 46	G 32
5	C 54	D 43	F 50
6	A 68	C 60	G 34
7	D 50	E 32	G 38

Source: W. J. Youden 1951. Statistical Methods for Chemists, John Wiley and Sons. Table 55 on page 104.

table 23 of chapter 4

Net Counting Rates for Radioactive Standards in Counts Per Second

	Sample			
Block	A	B	C	D
1	26.46	27.58	29.15	29.54
2	27.00	28.03	29.61	29.52
3	26.48	27.82	29.31	29.53
4	26.51	27.90	29.13	29.15

Source: H. H. Seliger 1950. A new method of radioactive standard calibration. Journal of Research of the National Bureau of Standards, 45: 496.

See also: W. J. Youden 1951. Statistical Methods for Chemists, page 88.

table 24 of chapter 4

Preferences Between Handwriting Specimens
(Entries Are % Favored Minus % Not Favored)

Minus specimen	Plus specimen								
	50a	50b	50c	70a	70b	70c	80a	80b	80c
a) CHICAGO DATA									
50a	0	4	34	90	98	96	98	94	88
50b	-4	0	20	70	90	92	96	96	90
50c	-34	-20	0	52	56	84	82	72	92
70a	-90	-70	-52	0	52	74	90	58	56
70b	-98	-90	-56	-52	0	48	60	4	42
70c	-96	-92	-84	-74	-48	0	18	-48	12
80a	-99	-96	-92	-90	-60	-18	0	-70	-38
80b	-97	-96	-72	-58	-4	+48	70	0	22
80c	-94	-90	-92	-56	-42	-12	38	-22	0
b) PRINCETON DATA									
50a	0	4	32	76	96	96	94	66	72
50b	-4	0	20	38	94	92	86	88	82
50c	-32	-20	0	40	74	88	84	68	86
70a	-76	-38	-40	0	56	68	82	40	66
70b	-96	-94	-64	-56	0	28	56	-26	22
70c	-96	-92	-88	-68	-28	0	42	-40	16
80a	-94	-86	-84	-82	-56	-42	0	-70	-24
80b	-68	-88	-68	-40	26	-40	70	0	40
80c	-72	-82	-86	-66	-22	-16	24	-40	0

Source: H. Gulliksen and J. W. Tukey 1958. Reliability for the law of comparative judgment. Psychometrika 23, 95-110, specifically table 1 on page 99.

each of 100 students, one at Chicago in the late 30's and one at Princeton in the late 40's. Analyze the two sets of data. Repeat this, omitting all entries of +80 or more and all entries of -80 or less. Contrast the results of the analysis.

7. Mosteller (1951) used as an example the relative game-winning of the teams of the American League in 1948. Table 25 gives the data. Analyze and comment.

8. Oesting and Kaufman (1945) presented data on the performance of a rapid method for the measurement of fat in meat. Table 26 gives some of the data. Analyze and comment.

9. Gore (1952) gives comparative exposure lives for 16 coating exposed four to a panel, both in Florida and in an accelerated aging device. Table 27 gives the data. Analyze this as best you can neglecting the differences in RELATIVE rates of aging in Florida and in X-1. Comment on your analyses and others that might be made.

table 25 of chapter 4

Fraction of Games Won Between Individual American League Teams in 1948

Losing team	Winning Team							
	Clev.	Bost.	N. Y.	Phil.	Detr.	St. L.	Wash.	Chic.
Clev.	-	.478	.545	.273	.409	.364	.273	.273
Bost.	.522	-	.364	.455	.318	.318	.318	.364
N. Y.	.455	.636	-	.455	.409	.273	.227	.273
Phil.	.727	.545	.545	-	.545	.182	.364	.273
Detr.	.591	.682	.591	.455	-	.500	.273	.364
St. L.	.636	.682	.727	.818	.500	-	.545	.381
Wash.	.727	.682	.773	.636	.727	.455	-	.429
Chic.	.727	.636	.727	.727	.636	.619	.571	-

Source: F. Mosteller 1951. Remarks on the method of paired comparisons. III. A test of significance for paired comparisons when equal standard deviations and equal correlations are assumed. Psychometrika 16: 207-218.

table 26 of chapter 4

% Fat in Meat by Standard and Rapid Methods

Sample	Meat	Standard method	Rapid method
1	Wieners	22.0	22.3
2	"	22.1	21.8
3	"	22.1	22.4
4	"	22.2	22.5
5	Chopped ham	24.6	24.9
6	Chopped pork	25.3	25.6
7	Chopped ham	25.3	25.8
8	"	25.6	26.2
9	"	25.6	26.1
10	"	25.9	26.7
11	Wieners	26.0	26.3
12	"	26.2	24.9
13	"	27.0	26.9
14	Chopped ham	27.3	28.4
15	"	27.7	27.1
16	Pork sausage links	41.5	41.4
17	"	41.6	41.4
18	Bulk pork sausage	45.5	45.5
19		48.5	48.2
20		49.1	47.5

Source: R. B. Oesting and I. P. Kaufman 1945. Industrial and Engineering Chemistry, Analytical Edition 17, 125.

See also: W. J. Youden 1947.

table 27 of chapter 4

Aging Results for 16 Surface Coatings
(Entries Give Coating Tag, Actual Life (in parenthesis),
Life as Fraction of Four Coating on Panel)

Panel	Loc.				
1	Fla	A (4 mo) .091	B (10 mo) .227	C (14 mo) .318	D (16 mo) .364
2	X-1	A (150 hr) .133	B (275 hr) .245	C (350 hr) .311	D (350 hr) .311
3	Fla	E (18 mo) .214	F (18 mo) .214	G (21 mo) .250	H (27 mo) .322
4	X-1	E (325 hr) .109	F (510 hr) .171	G (650 hr) .218	H (1500 hr) .502
5	Fla	I (10 mo) .182	J (21 mo) .382	K (15 mo) .273	L (9 mo) .163
6	X-1	I (625 hr) .234	J (900 hr) .336	K (700 hr) .262	L (450 hr) .168
7	Fla	M (24 mo) .363	N (8 mo) .118	O (12 mo) .176	P (24 mo) .353
8	X-1	M (1200 hr) .390	N (300 hr) .098	O (625 hr) .203	P (950 hr) .309
9	Fla	A (3 mo) .055	E (15 mo) .278	I (15 mo) .278	M (21 mo) .389
10	X-1	B (450 hr) .177	F (600 hr) .235	J (1000 hr) .392	N (500 hr) .196
11	Fla	C (12 mo) .200	G (24 mo) .400	K (18 mo) .300	O (6 mo) .100
12	X-1	D (250 hr) .093	H (1200 hr) .449	L (400 hr) .150	P (825 hr) .308